Published by Institute of Physics Publishing for SISSA/ISAS

Received: *March 24, 2003*
Revised: *May 29, 2003*
Accepted: *June 4, 2003*

Renormalization group and decoupling in curved space, II. The standard model and beyond

Eduard V. Gorbar

Departamento de Física, ICE, Universidade Federal de Juiz de Fora
MG, Brazil, and
Bogolyubov Institute for Theoretical Physics
Kiev, Ukraine
E-mail: gorbar@fisica.ufjf.br

Ilya L. Shapiro*

Departamento de Física, ICE, Universidade Federal de Juiz de Fora
MG, Brazil
E-mail: shapiro@fisica.ufjf.br

Abstract: We continue the study of the renormalization group and decoupling of massive fields in curved space, started in [1] and analyse the higher derivative sector of the vacuum metric-dependent action of the Standard Model. The QCD sector at low-energies is described in terms of the composite effective fields. For fermions and scalars the massless limit shows perfect correspondence with the conformal anomaly, but similar limit in a massive vector case requires an extra compensating scalar. In all three cases the decoupling goes smoothly and monotonic. A particularly interesting case is the renormalization group flow in the theory with broken supersymmetry, where the sign of one of the beta-functions changes on the way from the UV to IR.

Keywords: Renormalization Group, Physics of the Early Universe, Anomalies in Field and String Theories.

JHEP06(2003)004

*On leave from Tomsk State Pedagogical University, Tomsk, Russia.

Contents

1. Introduction

In the last decades there was a growing interest to the quantum effects of vacuum in curved space-time. In particular, there were investigations of the renormalization group flows for the parameters of the vacuum action in curved space-time: both perturbative and non-perturbative (see, e.g. the review [2] and further references therein). At present, the most of the non-perturbative results concern massless (mainly supersymmetric) models. In some cases, it was possible to establish the relation between the IR and UV β-functions for the parameters of the higher-derivative sector of the vacuum action.

An important application of the universal renormalization group flows in curved space is a candidate to be the natural model of inflation [3, 4], which is a modification of the well-known Starobinsky model [5, 6]. The main advantage of the modified Starobinsky model of inflation is that it is based on the first principles of quantum field theory and does not require conventional elements of the inflationary phenomenology such as inflaton, which do not have, at present, necessary particle physics justification. Moreover, this inflationary model does not require a special choice of the initial data and provides an automatic graceful exit to the FRW stage due to the supersymmetry breaking at low energies. The theoretical background of this approach to inflation consists of the quantum corrections to the classical action of vacuum [7, 8] treated in the effective Quantum Field Theory framework. One of the key points is the decoupling of heavy degrees of freedom (similar to the standard Appelquist and Carazzone theorem in flat space [9]) and especially sparticles. This decoupling results in the transition between the stable inflationary solution at the beginning and the automatic graceful exit to the FRW evolution at the end of the inflation epoch [4]. Qualitatively, the decoupling is a manifestation of the dominance of the mass terms in the quantum effects of the massive fields at low energies. In the gravitational case, the masses of the fields compete with the energies of the gravitational quanta at the external lines of the vacuum diagrams (see, e.g. [10]). Therefore, the quantitative study of the decoupling involves the analysis of the corresponding loop diagrams. Let us remark that, despite the decoupling theorem is well-known for the Quantum Field Theory in flat space-time (see, e.g. [11, 12]), the study of this phenomena in curved space-time has been started just recently in [1]. In this first paper we have investigated the massive scalar field non-minimally coupled to external gravity. The purpose of the present article is to generalize the results of [1] for the massive fermions and vectors, which are present in the low-energy spectrum in the electroweak (EW) and QCD sectors of the Standard Model of elementary particle physics (SM). Thus, we complete the description of the decoupling in the one-loop approximation. Furthermore, we shall discuss the decoupling of the sparticles in the supersymmetric theories beyond the SM. The present work should be considered as an important step to the future description of the behavior of the modified Starobinsky model in the interpolation regime between stable and unstable phases.

One of the problems in the study of the low-energy vacuum quantum effects of the SM is the non-perturbative nature of the low-energy QCD. Despite the existence of the mentioned above non-perturbative exact renormalization group flows in the massless supersymmetric version of QCD, they do not help too much in the realistic QCD theory which we are interested in here. As far as the non-perturbative results are not available for the phenomenologically relevant situations, we accept an effective field theory approach and describe the low-energy QCD using the Chiral Perturbation Theory model. It is well known, that all excitations of QCD at low energies are massive, and therefore their gravitational effects should be suppressed due to the decoupling mechanism. We are going to construct a qualitative description of this phenomena using the effective approach.

As we have seen in the previous work [1], the calculations on the flat background (or equivalent covariant calculations performed through the expansion in the powers of the curvature tensor) can not be conclusive for the description of the decoupling of the cosmological constant and inverse Newton constant, which is unaccessible within the usual perturbative approach. Therefore, in this paper we will consider the β-functions for the higher derivative terms in the effective action only. These terms are the most important ones for the inflationary model of [4], hence they deserve serious attention.

The paper is organized as follows. In section 2 we present a qualitative description of the low-energy QCD and the corresponding effective models in curved space. In section 3 the relation between the renormalization group for the massive fields and conformal anomaly for the corresponding massless fields is considered. Furthermore, we learn to make a distinction between various β-functions in the vacuum sector. In section 4 we derive the relevant expressions for the effective action of massive scalars, fermions and vectors in curved space-time. In the case of scalars and fermions the massless limits fit with the anomaly-induced effective action, while for the Proca field an extra compensating scalar is needed. In section 5 we discuss the one-loop renormalization group flows for these fields. The identification of the β-functions here is slightly different from the one in the previous paper [1]). In section 6 consider an application of these results for the model with broken supersymmetry. Finally, in section 7 we draw our conclusions.

2. Effective approach and the low-energy QCD in curved space

Our general purpose is to investigate the renormalization group flow between the UV and IR regimes in the SM in curved space-time. The method for investigating the decoupling which was developed in [1] is essentially based on the perturbative approach and the practical calculations can be successfully realized only in the one loop approximation. In the EW sector this approximation is reliable, because here we meet weakly interacting fields in both UV and IR regimes. Indeed, many of these fields gain mass at low energies due to Spontaneous Symmetry Breaking and Higgs mechanism, but it is clear that the general approach of [1] does not meet big obstacles here and should be sufficient for getting the necessary information.

Of course, the situation in the QCD sector of the SM is rather different. Due to the asymptotic freedom, in the UV regime the perturbative description works well and hence there is no difficulties with the use of the method of [1]. The real problem is the non-perturbative nature of QCD at low energies because, as we have just mentioned, our ability to make calculations in curved space-time is mainly restricted by the first order of the loop expansion. Hence, the first task is to understand which kind of information about the low-energy QCD in curved space one can obtain by making the perturbative calculations.

As well known, the QCD lagrangian in flat space-time reads

$$L = -\frac{1}{4} F_{\mu\nu}^{(a)} F^{(a)\,\mu\nu} + \sum_k \left(i\bar{\psi}_k \gamma_\mu \left(\partial_\mu + igT^a A_\mu^a \right) \psi_k + m_k \bar{\psi}_k \psi_k \right), \tag{2.1}$$

where $F_{\mu\nu}^{(a)} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f_{abc} A_\mu^b A_\nu^c$ is the field strength, T^a are the $SU_c(3)$ generators, f_{abc} are the structure constants of the SU(3) algebra, and the summation is performed over the quark flavors. Due to the gauge symmetry the mass terms for gauge vector bosons are prohibited such that the gluons are massless at the lagrangian level. Switching on the gravitational field, one may ask whether the gluons contribute to the running of the parameters of the gravitational action (3.1) at low energies, including in the present Universe?[1] In what follows we argue that the answer to this question is definitely no. Let us remember that, below the nucleosynthesis scale, the magnitude of the Hubble parameter is much smaller than even the masses of the lightest neutrinos [10]. Consequently, if we

[1]We thank Natan Berkovits for this question which motivated the discussion of this section.

restrict our consideration to the semiclassical approach and therefore do not consider the quantum gravity effects, then only the photon and gluons may, in principle, contribute to the running.

In order to study the QCD contribution to the vacuum gravitational renormalization group in the present Universe, first of all we need to know how the QCD excitation spectrum gets modified when we go from the high energy scale down to the very low energies. At high energies the QCD coupling constant is small and the perturbation theory based on the gluons and quarks is meaningful. Contrary to that, the low energy QCD, in terms of quarks and gluons (2.1), is non-perturbative. At present, it is not possible to determine the low-energy QCD spectrum in a purely theoretical framework, starting from the first principles, and we need a certain phenomenological input here. Fortunately, the QCD excitation spectrum at low energies is known from experiment [13]. It is well known that there are no particles with the quantum numbers of gluons (needless to say that to explain theoretically the absence of gluons and quarks in the low energy QCD spectrum is a part of the confinement problem), and only composite particles like pions and kaons are observed. Perhaps, it is worth reminding that due to the unbroken Poincare symmetry all states in the low-energy QCD excitation spectrum are characterized by their mass and spin and as usual one interpret these states as particles.

And so, what we really need is the effective spectrum of the low-energy QCD in terms of the composite particles. Presumably, the lightest states are the most important ones, because they should decouple from gravity at lower energies. According to the Particle Data Group [13], the lowest QCD excitations are the octet of pseudoscalar mesons: the pions π^0 (135 MeV) and $\pi^{\pm}$ (139.6 MeV), the kaons $K^{\pm}$ (493.7 MeV), $\bar{K}^0$, and K^0 (497.7 MeV), and the η-meson η (547.3 MeV). In massless QCD (when all bare quark masses are zero), these mesons would be massless since they are the Goldstone bosons connected with the Chiral Symmetry Breaking (therefore, these mesons are pseudoscalars). The nine would be Goldstone boson η' acquires a nonzero mass due to the $U_A(1)$ axial anomaly. Since chiral symmetry is explicitly broken in the real QCD, the pions, kaons, and the η-meson acquire nonzero masses. For example, the $\pi^{\pm}$ mass is given by

$$m_{\pi^{\pm}}^2 = -\frac{m_u + m_d}{f_\pi^2}\langle \bar{u}u\rangle \,,$$

where m_u and m_d are the u- and d-quark masses, f_π is the pion decay constant, and $\langle \bar{u}u\rangle$ is the quark condensate. In addition to the pseudoscalar mesons, there are also some relatively light scalar in the QCD spectrum like f_0 (600 MeV) and a_0 (980 MeV), however, these scalar states are difficult to resolve because of their large decay widths. The vector particle with the lowest mass is the ρ meson. Although its mass ρ (771.1 MeV) is larger than pseudoscalars masses, the ρ meson plays a very important role in particles interactions of the low-energy QCD. Let us mention here only the Vector Dominance and vector meson universality advocated by Sakurai [14].

The fermions with lowest masses in QCD are nucleons $m_p = 938.3\, MeV$ and $m_n = 939.6\, MeV$. Note that their masses are significantly larger than that of the pseudoscalar mesons. Let us remark that the decoupling is one of the key ideas of the Heavy Baryon

Chiral Perturbation Theory, for it allows the perturbative study of the interaction of nucleons and pions despite the large value of the pion-nucleon coupling constant. Of course, this approach is possible only at low energies (for details, see, e.g., [15]).

According to the Chiral Perturbation Theory, the pions and kaons are the would be Goldstone bosons because the chiral symmetry is explicitly broken. Therefore, their interaction can be represented [16, 17] as a series in $p^2/16\pi^2 f_\pi^2$ (where p is a particle momentum), $m_\pi^2/16\pi^2 f_\pi^2 \approx 0.01$, and $m_K^2/16\pi^2 f_\pi^2 \approx 0.18$. It is convenient to collect the pseudoscalar fields in a unitary 3×3 matrix

$$U = \exp\left(\frac{i\sqrt{2}\Phi}{f_\pi}\right), \qquad \Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ *\pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ *K^- & \overline{K^0} & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

and the effective lagrangian of Chiral Perturbation Theory is then constructed as a series in derivatives of U and the pseudoscalars masses. For example, the leading-order term of the effective lagrangian is given by

$$L_2 = \frac{f_\pi}{4}\,\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger + \chi U^\dagger + U\chi^\dagger\right), \tag{2.3}$$

where the matrix χ defines the pseudoscalars masses. The next-to-leading order effective lagrangian contains terms with four derivatives, terms quadratic in derivatives and linear in χ, and terms quadratic in χ.

Thus, for momenta less than $4\pi f_\pi$, the interaction of pions and kaons is weak and, consequently, it is a reasonably good approximation to consider one loop diagrams with free pions, kaons, and η-mesons in order to study the QCD contribution to the renormalization group running of the parameters of the vacuum gravitational action at low energies. Finally, since QCD does not have massless particles in its excitation spectrum and at energies less than m_{π^0} all QCD particles decouple, the QCD contribution to the renormalization group running of the gravitational action parameters is suppressed at low energies. In order to verify this, in a first approximation, it is sufficient to analyse the contributions of free massive scalars, spinors and vectors to the gravitational effective action at low energies.

3. Renormalization group and conformal anomaly

According to the previous section, the investigation of decoupling of the quantized massive fields from an external gravity may be reduced to the derivation of the effective action of vacuum for those fields: scalars, fermions, and vecctors. In the IR, we expect that the mass terms will dominate and this leads to the low-energy decoupling. At high energies, we expect that the effects of the masses become negligible such that the effective action in the limit $m \to 0$ coincides with the effective action derived for massless fields. In fact, this expectation is completely justified for the scalar and spinor fields and not justified at all for the massive vector. The reason is that the Proca field has larger number of physical

JHEP06(2003)004

degrees of freedom than the massless gauge vector field. This can be seen explicitly if one uses the Boulware transformation [18] for the massive vector or apply an equivalent one-loop procedure described in [19]. In both cases one can see that the Proca field has one extra scalar degree of freedom compared to the gauge boson.

In the MS scheme, the metric-dependent vacuum divergences must be removed by adding appropriate local counterterms and renormalizing parameters $1/G$, Λ, $a_{1,2,3,4}$ of the classical action of vacuum

$$S_{vac} = \int d^4x\sqrt{-g}\Big\{ -\frac{1}{16\pi G}(R+2\Lambda) + a_1 C^2 + a_2 E + a_3 \Box R + a_4 R^2 \Big\}\,, \tag{3.1}$$

where

$$C^2 = R^2_{\mu\nu\alpha\beta} - 2R^2_{\alpha\beta} + 1/3\, R^2 \tag{3.2}$$

is the square of the Weyl tensor and

$$E = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4\, R_{\alpha\beta}R^{\alpha\beta} + R^2 \tag{3.3}$$

is the integrand of the Gauss-Bonnet topological invariant. Let us consider the massless conformal invariant fields in the one-loop approximation. In this case only the higher derivative conformal part of the vacuum action (including the surface and topological terms)

$$S_{HDC} = \int d^4x\sqrt{-g}\,\left[a_1 C^2 + a_2 E + a_3 \Box R\right]\,, \tag{3.4}$$

is subject to renormalization. S_{HDC} can be called "conformal", because it satisfies the Noether identity

$$-\frac{2}{\sqrt{-g}}g_{\mu\nu}\frac{\delta S_{HDC}}{\delta g_{\mu\nu}} = 0\,. \tag{3.5}$$

On quantum level, the classical action of vacuum gains loop corrections and must be replaced by the corresponding effective action. Traditionally, the violation of the conformal identity (3.5) at quantum level is called the conformal anomaly of the Energy-Momentum (stress) tensor. This anomaly has the form (see, e.g. [8] and references therein)

$$\langle T^\mu_\mu\rangle = \beta_1^{\overline{MS}} C^2 + \beta_2^{\overline{MS}} E + \beta_3^{\overline{MS}}\, \Box R\,, \tag{3.6}$$

where $\beta_1^{\overline{MS}}$, $\beta_2^{\overline{MS}}$ and $\beta_3^{\overline{MS}}$ are the minimal subtraction ($\overline{MS}$-scheme) based β-functions of the parameters $a_{1,2,3}$ correspondingly (see e.g [31])

$$\begin{aligned}
\beta_1^{\overline{MS}} &= -\frac{1}{(4\pi)^2}\left(\frac{1}{120}N_0 + \frac{1}{20}N_{1/2} + \frac{1}{10}N_1\right)\,,\\
\beta_2^{\overline{MS}} &= \frac{1}{(4\pi)^2}\left(\frac{1}{360}N_0 + \frac{11}{360}N_{1/2} + \frac{31}{180}N_1\right)\,,\\
\beta_3^{\overline{MS}} &= -\frac{1}{(4\pi)^2}\left(\frac{1}{180}N_0 + \frac{1}{30}N_{1/2} - \frac{1}{10}N_1\right)\,.
\end{aligned} \tag{3.7}$$

JHEP06(2003)004

Here $N_0, N_{1/2}$ and N_1 are the numbers of scalar, fermion, and vector massless fields, respectively. Let us remark that the β-functions $\beta_2^{\overline{MS}}$ and $\beta_3^{\overline{MS}}$ result from the renormalization of the topological and surface terms in (3.1). The corresponding divergences can be obtained explicitly using, for example, the Schwinger-DeWitt formalism (see, e.g. [8]). However, in other calculational schemes, like the one which we shall apply below, topological and surface divergences are not seen explicitly. Therefore, these two β-functions may be identified only through the analysis of the finite part of the effective action which is related to the anomaly (3.6).

In the massive case, the higher-derivative divergences are exactly the same as for the massless fields. In the following sections we shall apply the physical mass-dependent scheme of renormalization and derive corresponding physical β-functions which are generally different from the ones of (3.7). In order to distinguish the two kinds of the renormalization group functions, we shall denote the $\overline{MS}$-scheme β-functions as $\beta^{\overline{MS}}$. The UV and IR limits of the β-functions derived in the mass-dependent scheme will be denoted as β^{UV} and β^{IR}. Of course, we expect that the correctly defined β-function would satisfy the relation

$$\beta^{UV} = \beta(\overline{\mathrm{MS}}) + \mathcal{O}\left(\frac{m^2}{p^2}\right).$$

In this paper we shall consider β_1, β_3 and β_4, corresponding to the renormalization of the parameters a_1, a_3 and a_4 of the vacuum action (3.1). The analysis of β_2 will not be presented here, because it requires much more involved calculation in the third order in curvature [20]. Moreover, the decoupling in the β_2-sector is less important for the application of the renormalization group and anomaly to inflation [4].

For the simple situation with the massless fields, the expression (3.6) enables one to derive, in an explicit and economic way, the effective action of gravity which is exact for the particular case of the homogeneous and isotropic FRW metric. The non-local covariant form of the anomaly-induced effective action is [21, 22]

$$\begin{aligned}\Gamma_{\mathrm{ind}} = S_c[g_{\mu\nu}] &+ \frac{3\beta_3^{\overline{MS}} + 2\beta_2^{\overline{MS}}}{36}\int d^4x\sqrt{-g(x)}R^2(x) - \\ &- \int d^4x\sqrt{-g(x)}\int d^4y\sqrt{-g(y)}\left[E - \frac{2}{3}\Box R\right]_x \times \\ &\times G(x,y)\left[\frac{\beta_1^{\overline{MS}}}{4}C^2 + \frac{\beta_2^{\overline{MS}}}{8}(E - \frac{2}{3}\Box R)\right]_y .\end{aligned} \tag{3.8}$$

The conformal invariant functional $S_c[g_{\mu\nu}]$ is an integration constant which can not be obtained using the conformal anomaly. The effective action (3.8) contains a Green function $G(x,y)$ of the conformal differential fourth order scalar operator

$$\Delta_4 = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^\mu R)\nabla_\mu . \tag{3.9}$$

The local term $\int d^4x\sqrt{-g}R^2$ in the first line of (3.8) gains contributions from two β-functions $\beta_2^{\overline{MS}}$ and $\beta_3^{\overline{MS}}$. But, as we shall see in a moment, for our purposes only the $\beta_3^{\overline{MS}}$-dependent contribution is relevant. As it was already mentioned, we are going to use the massless effective action (3.8) in order to check the more general expression for the effective action of the massive fields in the UV limit. But, the calculations for the massive case will be performed only in the second order in curvature, so that in order to compare the two results we have to expand (3.8) up to the second order in curvature. In the situation of interest, this is equivalent to the bilinear expansion in the metric perturbation

$$h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$$

on the flat background [1]. In both cases the operator (3.9) becomes just $\Box^2$. The terms with C^2 and E in the second line of eq. (3.8) can be safely neglected, for they are, at least, of the third order. Then, the remaining term in the second line of (3.8) is

$$\frac{\beta_2^{\overline{MS}}}{18} \int d^4x\sqrt{-g(x)} \int d^4y\sqrt{-g(y)}(\Box R)_x \left(\frac{1}{\Box}\right)_{(x,y)} (\Box R)_y \,.$$

After some partial integrations, this terms precisely cancels with the $\beta_2^{\overline{MS}}$-dependent term in the first line of (3.8). Thus, the important consequence of our consideration is that, in order to perform verification of the second-order in curvature calculations for the massive case, we need to take only the $\beta_3^{\overline{MS}}$-dependent part of the $\int d^4x\sqrt{-g}R^2$-term in (3.8)

$$\Gamma_{\beta_3} = \frac{1}{12(4\pi)^2} \int d^4x\sqrt{-g} \left(-\frac{1}{180}N_0 - \frac{1}{30}N_{1/2} + \frac{1}{10}N_1\right) R^2 \,, \tag{3.10}$$

Also, as a by-product we can see whether the quantum correction (3.8) contributes to the propagation of the gravitational waves in the flat space, e.g. in the modern Universe. Let us notice that the $\int \sqrt{-g}R^2$ term does not contribute to the propagation of the transverse traceless (that is spin-2) mode. Therefore, the contribution to the equation for the gravitational wave on the flat background may come only from the conformal invariant functional S_c but not from the anomaly-induced part.

There is another aspect of the $\int d^4x\sqrt{-g}R^2$-term, which is important for us. The anomaly-induced effective actions correspond to the massless conformal fields. However, there is an example of the field which is massless but not conformal: the scalar one with the non-minimal parameter $\xi \neq 1/6$. In this case the parameter a_4 in the vacuum action (3.1) has to be renormalized independently and there is a corresponding β-function $\beta_4^{\overline{MS}}$. On the other hand, the $\Box R$-term in the conformal anomaly produces the finite $\int \sqrt{-g}R^2$-term in the effective action (3.8). For the massless case, these two R^2-terms do not mix, because the renormalization group is related to the divergent part of the effective action only. For the massive fields, in a physical mass-dependent scheme, the situation is quite different, because the β-functions result from the finite part of the effective action. Then, the division of the $\int \sqrt{-g}R^2$-term in the effective action between two β-functions: β_3 for the a_3 parameter and β_4 for the a_4 parameter in the action (3.1) becomes ambiguous and one has to define the sharing in some appropriate way.

In the previous work [1] we attributed all the $\int \sqrt{-g}R^2$-term to the β_4. In the present paper we use another definition, which looks more natural. Since the infinite $\int \sqrt{-g}R^2$-type counterterm is absent for $\xi = 1/6$, we include all the terms proportional to $(\xi - 1/6)$

into β_4-function. All other terms will be attributed to the β_3. Let us remark that this form of the sharing is the most useful for the cosmological application [4], because the existing version of the modified Starobinsky model is based on the supposition that $\xi \approx 1/6$. Another advantage of this definition is the following. We expect that in the massless limit the vacuum effective action of massive fields will converge to the anomaly-effective action. But, it is well known that the anomaly can be integrated only if the R^2-term in the anomalous trace is absent [21]. In turn, this requires the absence of the $\int \sqrt{-g}R^2$-counterterm, that can be achieved for the $\xi = 1/6$ only. Hence, our way to identify the two β-functions is natural in the sense it helps making the massless limit look simpler.

4. The covariant derivation of the effective action up to the second order in curvature

In this section we perform the derivation of the second order in curvature $\mathcal{O}(R^2)$-terms in the one-loop effective action using the general expression for the heat kernel of the differential second order operator derived in [20, 23]. In the previous article [1] we have demonstrated, using the massive scalar field as an example, that this covariant heat-kernel approach is completely equivalent to the calculation of the quantum correction to the graviton propagator from the matter loop (see also [24] for the similar calculation). Hence, our approach is analogous to the standard study of decoupling in QED [12].

We define the one-loop euclidean effective action of a field with mass m as a trace of an integral of the heat kernel over the proper time s (compared to [20] there is an important $\exp[-m^2 s]$ insertion)

$$\bar{\Gamma}^{(1)} = -\frac{1}{2}\mathrm{Tr}\ln\left(-\nabla^2\hat{1} + m^2 - \hat{P} + \frac{1}{6}R\hat{1}\right) = -\frac{1}{2}\int_0^\infty \frac{ds}{s}\,\mathrm{tr}\,K(s)\,, \tag{4.1}$$

where $K(s)$ is the heat kernel

$$\begin{aligned}\mathrm{tr}\,K(s) = {} & \frac{(\mu^2)^{2-w}}{(4\pi s)^w}\int d^4x g^{1/2} e^{-sm^2} \times \\ & \times\,\mathrm{tr}\Big\{\hat{1} + s\hat{P} + s^2\Big[R_{\mu\nu}f_1(-s\nabla^2)R^{\mu\nu} + +Rf_2(-s\nabla^2)R + \hat{P}f_3(-s\nabla^2)R \\ & + \hat{P}f_4(-s\nabla^2)\hat{P} + \hat{\mathcal{R}}_{\mu\nu}f_5(-s\nabla^2)\hat{\mathcal{R}}^{\mu\nu}\Big]\Big\} + \mathcal{O}(\mathcal{R}^3)\,, \end{aligned}\tag{4.2}$$

where $\hat{\mathcal{R}}_{\mu\nu} = \Big[\nabla_\mu, \nabla_\nu\Big]$ is a commutator of covariant derivatives in the space of the fields of interest. The functions $f_{1,2,..,5}(\tau)$ are given by the following expressions:

$$\begin{aligned} f_1(\tau) &= \frac{f(\tau) - 1 + \tau/6}{\tau^2}\,, \qquad f_2(\tau) = \frac{f(\tau)}{288} + \frac{f(\tau)-1}{24\tau} - \frac{f(\tau) - 1 + \tau/6}{8\tau^2}\,, \\ f_3(\tau) &= \frac{f(\tau)}{12} + \frac{f(\tau)-1}{2\tau}\,, \qquad f_4(\tau) = \frac{f(\tau)}{2}\,, \qquad f_5(\tau) = \frac{1-f(\tau)}{2\tau}\,, \end{aligned}$$

where

$$f(\tau) = \int_0^1 d\alpha\, e^{\alpha(1-\alpha)\tau}\,, \qquad \tau = -s\nabla^2\,.$$

Below we consider how these formulas can be applied to the derivation of the effective action of scalars, fermions (in this case the overall sign in the eq. (4.1) has to be changed, of course), and vectors.

4.1 Massive scalar with the non-minimal coupling

Let us first repeat the calculation for the scalar field performed in the previous paper [1]. The main difference is that here we perform more detailed analysis of the massless UV limit for the non-conformal case $\xi \neq 1/6$.

For the massive real scalar with the non-minimal coupling to gravity

$$\hat{P} = -\left(\xi - \frac{1}{6}\right) R \qquad \text{and} \qquad \hat{\mathcal{R}}_{\mu\nu} = 0\,. \tag{4.3}$$

Introducing the new variable $sm^2 = t$ and notation $u = \tau/t$, we arrive at the following integral representation for the effective action:

$$\begin{aligned}
\bar{\Gamma}^{(1)} &= -\frac{1}{2(4\pi)^2}\int d^4x g^{1/2}\left(\frac{m^2}{4\pi\mu^2}\right)^{w-2} \times \\
&\times \int_0^\infty dte^{-t}\left\{\frac{m^4}{t^{w+1}} + \left(\xi - \frac{1}{6}\right)\frac{Rm^2}{t^w} + \sum_{i=1}^{5} l_i^* \cdot R_{\mu\nu}M_iR^{\mu\nu} + \sum_{j=1}^{5} l_j \cdot RM_jR\right\},
\end{aligned} \tag{4.4}$$

where

$$\begin{aligned}
&l_{1,2}^* = 0\,, \qquad l_3^* = 1\,, \qquad l_4^* = \frac{1}{6}\,, \qquad l_5^* = -1\,;\\
&l_1 = \frac{1}{288} - \frac{1}{12}\left(\xi - \frac{1}{6}\right) + \frac{1}{2}\left(\xi - \frac{1}{6}\right)^2, \qquad l_2 = \frac{1}{24} - \frac{1}{2}\left(\xi - \frac{1}{6}\right),\\
&l_3 = -\frac{1}{8}\,, \qquad l_4 = -\frac{1}{16} + \frac{1}{2}\left(\xi - \frac{1}{6}\right), \qquad l_5 = \frac{1}{8}
\end{aligned}$$

and

$$M_1 = \frac{f(tu)}{u^2t^{w+1}}\,, \qquad M_2 = \frac{f(tu)}{ut^w}\,, \qquad M_3 = \frac{f(tu)}{u^2t^{w+1}}\,, \qquad M_4 = \frac{1}{ut^w}\,, \qquad M_5 = \frac{1}{u^2t^{w+1}}\,. \tag{4.5}$$

As we shall see later on, the representation (4.4) exists also for the fermion and vector fields. The difference is just the values of the coefficients l_i^* and l_j. Hence, from practical point of view it is better to derive the integrals $\int_0^\infty dt\, e^{-t} M_i(t,u)$ only once and later use them as a standard reference.

Following Barvinsky and Vilkovisky [20], we adopt the dimensional regularization in the form suggested by Brown and Cassidy [25] (see also [19] for useful technical details). The UV limit $\tau/sm^2 \gg 1$ and the IR limit $\tau/sm^2 \ll 1$ can be easily investigated in this

JHEP06(2003)004

framework. The relevant integrals are

$$
\begin{aligned}
&\left(\frac{m^2}{\mu^2}\right)^{\omega-2}\int_0^\infty \frac{dt}{(4\pi)^\omega}M_1(t)=\left(\frac{m^2}{4\pi\mu^2}\right)^{w-2}\int_0^\infty \frac{dt}{(4\pi)^2}e^{-t}t^{1-\omega}\int_0^1 d\alpha e^{\alpha(1-\alpha)tu}=\\
&\quad=\frac{1}{(4\pi)^\omega}\left[\frac{1}{2-\omega}+\ln\left(\frac{m^2}{4\pi\mu^2}\right)+2A\right], \qquad (4.6)\\
&\left(\frac{m^2}{\mu^2}\right)^{\omega-2}\int_0^\infty \frac{dt}{(4\pi)^\omega}M_2(t)=\left(\frac{m^2}{4\pi\mu^2}\right)^{w-2}\int_0^\infty \frac{dt}{(4\pi)^2}e^{-t}t^{-\omega}\frac{1}{u}\int_0^1 d\alpha e^{\alpha(1-\alpha)tu}=\\
&\quad=\frac{1}{(4\pi)^\omega}\left\{\left[\frac{1}{2-\omega}+\ln\left(\frac{m^2}{4\pi\mu^2}\right)+1\right]\cdot\left(\frac{1}{12}-\frac{1}{a^2}\right)-\frac{4A}{3a^2}+\frac{1}{18}\right\}, \qquad (4.7)\\
&\left(\frac{m^2}{\mu^2}\right)^{\omega-2}\int_0^\infty \frac{dt}{(4\pi)^\omega}M_3(t)=\left(\frac{m^2}{4\pi\mu^2}\right)^{w-2}\int_0^\infty \frac{dt}{(4\pi)^2}e^{-t}t^{-1-\omega}\frac{1}{u^2}\int_0^1 d\alpha e^{\alpha(1-\alpha)tu}=\\
&\quad=\frac{1}{(4\pi)^\omega}\left\{\left[\frac{1}{2-\omega}+\ln\left(\frac{m^2}{4\pi\mu^2}\right)+\frac{3}{2}\right]\times\right.\\
&\qquad\qquad\left.\times\left(\frac{1}{2a^4}-\frac{1}{12a^2}+\frac{1}{160}\right)+\frac{8A}{15a^4}-\frac{7}{180a^2}+\frac{1}{400}\right\}, \qquad (4.8)\\
&\left(\frac{m^2}{\mu^2}\right)^{\omega-2}\int_0^\infty \frac{dt}{(4\pi)^\omega}M_4(t)=\left(\frac{m^2}{4\pi\mu^2}\right)^{w-2}\int_0^\infty \frac{dt}{(4\pi)^2}e^{-t}t^{-\omega}\frac{1}{u}=\\
&\quad=\frac{1}{(4\pi)^\omega}\cdot\left[\frac{1}{2-\omega}+\ln\left(\frac{m^2}{4\pi\mu^2}\right)+1\right]\cdot\left(\frac{1}{4}-\frac{1}{a^2}\right), \qquad (4.9)
\end{aligned}
$$

and

$$
\begin{aligned}
\left(\frac{m^2}{\mu^2}\right)^{\omega-2}\int_0^\infty \frac{dt}{(4\pi)^\omega}M_5(t) &= \left(\frac{m^2}{4\pi\mu^2}\right)^{w-2}\int_0^\infty \frac{dt}{(4\pi)^2}e^{-t}t^{-1-\omega}\frac{1}{u^2} \qquad (4.10)\\
&= \frac{1}{(4\pi)^\omega}\cdot\left[\frac{1}{2(2-\omega)}+\frac{1}{2}\ln\left(\frac{m^2}{4\pi\mu^2}\right)+\frac{3}{4}\right]\cdot\left(\frac{1}{4}-\frac{1}{a^2}\right)^2,
\end{aligned}
$$

where

$$
A=-\frac{1}{2}\int_0^1 d\alpha\ln\Big[1+\alpha(1-\alpha)u\Big]=1-\frac{1}{a}\ln\frac{1+a/2}{1-a/2} \quad \text{and} \quad a^2=\frac{4\nabla^2}{\nabla^2-4m^2}\,. \qquad (4.11)
$$

In the expressions (4.6)–(4.10) we disregarded those terms which vanish in the $\omega\to 2$ limit. Now one has to replace these integrals into eq. (4.4) and use the relation [20][2]

$$
R_{\mu\nu}\widehat{X}R^{\mu\nu}-\frac{1}{3}R\widehat{X}R=\frac{1}{2}C_{\mu\nu\alpha\beta}\widehat{X}C^{\mu\nu\alpha\beta}+\mathcal{O}(R^3)\,, \qquad (4.12)
$$

valid for any operator $\widehat{X}$ build up from the powers of the covariant derivative. In this way, we arrive at the effective action for massive nonminimal scalar in the $\mathcal{O}(R^2)$ approxima-

[2]This relation holds only in the second order in curvature, and also means that we disregard the topological Gauss-Bonnet term. In other words, it can be used if we are interested in the corrections to the gravitational propagator, but not to the vertex terms.

tion [1]

$$\bar{\Gamma}^{(1)}_{\rm scalar} = \frac{1}{2(4\pi)^2}\int d^4x g^{1/2}\times$$
$$\times\Bigg\{\frac{m^4}{2}\cdot\left[\frac{1}{2-w}+\ln\left(\frac{4\pi\mu^2}{m^2}\right)+\frac{3}{2}\right]+\left(\xi-\frac{1}{6}\right)m^2R\left[\frac{1}{2-w}+\ln\left(\frac{4\pi\mu^2}{m^2}\right)+1\right]+$$
$$+\frac{1}{2}C_{\mu\nu\alpha\beta}\left[\frac{1}{60(2-w)}+\frac{1}{60}\ln\left(\frac{4\pi\mu^2}{m^2}\right)+k_W(a)\right]C^{\mu\nu\alpha\beta}+$$
$$+R\left[\frac{1}{2}\left(\xi-\frac{1}{6}\right)^2\left(\frac{1}{2-w}+\ln\left[\frac{4\pi\mu^2}{m^2}\right]\right)+k_R(a)\right]R\Bigg\}, \tag{4.13}$$

where

$$k_W(a) = \frac{8A}{15a^4}+\frac{2}{45a^2}+\frac{1}{150},$$
$$k_R(a) = A\left(\xi-\frac{1}{6}\right)^2-\frac{A}{6}\left(\xi-\frac{1}{6}\right)+\frac{2A}{3a^2}\left(\xi-\frac{1}{6}\right)+\frac{A}{9a^4}-\frac{A}{18a^2}+\frac{A}{144}+$$
$$+\frac{1}{108a^2}-\frac{7}{2160}+\frac{1}{18}\left(\xi-\frac{1}{6}\right). \tag{4.14}$$

An important check of the expression (4.13) can be performed through the massless limit $m\to 0$, where we expect to meet the $\mathcal{O}(R^2)$ part of the anomaly-induced effective action (3.8). It is easy to see that the limit $m\to 0$ is singular for the expression A in (4.11). Fortunately, for $\xi=1/6$, the A-dependent terms which contain this singularity cancel and we obtain

$$\bar{\Gamma}^{(1)}(\xi=1/6, m\to 0) = -\frac{1}{12\cdot 180(4\pi)^2}\int d^4x g^{1/2}R^2+\cdots. \tag{4.15}$$

This is exactly what we should expect due to (3.10). Hence, the free scalar field with $\xi=1/6$ has consistent UV limit. But, let us remind that the nonminimal parameter ξ can not be equal to $1/6$ precisely within the interacting theory, because at higher loops such theory would be non-renormalizable [26]. On the other hand, if we set $\xi\neq 1/6$ and perform the massless limit, the overall $(\xi-1/6)^2\int\sqrt{-g}R^2$ term in the effective action has regular behavior due to the cancelation with the $\ln\left[4\pi\mu^2/m^2\right]$ term. The singularity really appears not in the $m^2\to 0$ limit, but in the high-energy $|p^2|\to\infty$ limit. Of course, the coefficient of this divergence is identical to the pole $1/(\omega-2)$ coefficient. Hence, the singularity of the expression A in the $m^2\to 0$ or $|p^2|\to\infty$ limits is related to the UV divergence and must be treated by renormalization. Technically, this singularity shows the relation between the divergent and finite parts of the effective action in the UV. Similarly, in the massless limit $a\to 2$ there are terms proportional to A in the Weyl term formfactor $k_W(a)$. The UV singularities in $k_W(a)$ and $k_R(a)$ for $\xi\neq 1/6$ mean the appearance of the covariant non-local terms like

$$\int d^4x\sqrt{-g}C_{\mu\nu\alpha\beta}\ln\left(-\frac{\Box}{\mu^2}\right)C_{\mu\nu\alpha\beta}\qquad\text{and}\qquad\int d^4x\sqrt{-g}R\ln\left(-\frac{\Box}{\mu^2}\right)R\,. \tag{4.16}$$

These terms correspond to the renormalization group running, which we are going to discuss in section 5.

4.2 Massive fermion theory

In this case the differential operator of interest is

$$\hat{H}_f = i\left(\gamma^\mu \nabla_\mu + i m_f\right), \tag{4.17}$$

where m_f is a fermion mass. After the standard doubling procedure (see, e.g. [8] and also [27] for a proof of the equality $\mathrm{Tr}\ln[\gamma^\mu \nabla_\mu + i m_f] = \mathrm{Tr}\ln[\gamma^\mu \nabla_\mu - i m_f]$) and using the relations

$$\hat{\mathcal{R}}_{\mu\nu}\psi = \Big[\nabla_\mu, \nabla_\nu\Big]\psi = -\frac{1}{4} R_{\mu\nu\alpha\beta}\gamma^\alpha\gamma^\beta\psi \qquad \text{and} \qquad \gamma^\mu\gamma^\nu\nabla_\mu\nabla_\mu = \nabla^2 - \frac{1}{4}R\,,$$

we arrive at the following coefficients in the massive fermion case:

$$l_1 = 0\,, \qquad l_2 = -\frac{1}{16}\,, \qquad l_3 = -\frac{1}{8}\,, \qquad l_4 = \frac{1}{24}\,, \qquad l_5 = \frac{1}{8}\,. \tag{4.18}$$

$$l_1^* = 0\,, \qquad l_2^* = \frac{1}{4}\,, \qquad l_3^* = 1\,, \qquad l_4^* = -\frac{1}{12}\,, \qquad l_5^* = -1\,. \tag{4.19}$$

Of course, the sign of the whole expression (4.2) must be changed due to the fermion statistics of the field. After we established the coefficients (4.18) and (4.19), the calculation reduces to the routine application of the equations (4.6)–(4.10). The effective action has the form

$$\begin{aligned}
\bar{\Gamma}^{(1)}_{\text{fermion}} = {} & \frac{1}{2(4\pi)^2}\int d^4x g^{1/2} \times \\
& \times \Bigg\{ -2m_f^4 \cdot \left[\frac{1}{2-w} + \ln\left(\frac{4\pi\mu^2}{m_f^2}\right) + \frac{3}{2}\right] + \frac{1}{3}m_f^2 R \cdot \left[\frac{1}{2-w} + \ln\left(\frac{4\pi\mu^2}{m_f^2}\right) + 1\right] + \\
& + \frac{1}{2}C_{\mu\nu\alpha\beta}\left[\frac{1}{10(2-w)} + \frac{1}{10}\ln\left(\frac{4\pi\mu^2}{m_f^2}\right) + k_W^f(a)\right] C^{\mu\nu\alpha\beta} + R\left[k_R^f(a)\right] R \Bigg\}\,,
\end{aligned} \tag{4.20}$$

where

$$\begin{aligned}
k_W^f(a) &= \frac{300Aa^2 - 480A - 40a^2 + 19a^4}{225a^4}\,, \\
k_R^f(a) &= \frac{a^4 - 120A - 10a^2 + 30Aa^2}{270a^4}\,.
\end{aligned} \tag{4.21}$$

Of course, in the expressions for a and A from (4.11), one has to replace the scalar mass m by the fermion one m_f. Let us remark that the R^2-type divergence in (4.20) is absent, because the divergent term does not depend on the presence of a mass and because the massless theory is conformally invariant.

Finally, in the massless limit $m_f \to 0$ we obtain

$$\bar{\Gamma}^{(1)}(m_f \to 0) = -\frac{1}{12 \cdot 30(4\pi)^2}\int d^4x g^{1/2} R^2 + \cdots\,, \tag{4.22}$$

in a perfect correspondence with the expected result (3.10).

4.3 Massive vector theory

The massive vector case (Proca theory) is the most complicated one, this especially concerns the correspondence with the anomaly-induced result (3.10) in the massless limit. Let us remark that this correspondence represents a very complete test of the calculations. The resulting $\int \sqrt{-g}R^2$-term is sensible to the contributions from absolutely all higher derivative terms in the effective action for the massive theory. Those terms which do not contribute to (3.10) directly are verified through the cancelation of the singular A-structures or through the correspondence with the pole terms.

The massive vector operator is

$$\hat{H}_v = \delta^\mu_\nu \Box - \nabla^\mu \nabla_\nu - R^\mu_\nu - \delta^\mu_\nu m_v^2 \,, \tag{4.23}$$

and one can show that [19]

$$\mathrm{Tr}\ln \hat{H}_v = \mathrm{Tr}\ln \left(\delta^\mu_\nu \Box - R^\mu_\nu - \delta^\mu_\nu m_v^2\right) - \mathrm{Tr}\ln \left(\Box - m_v^2\right) + \ln m^2 \int d^4x \delta^\mu_\mu \delta(0) \,, \tag{4.24}$$

where the first term is the non-degenerate massive vector operator and the second term is a scalar operator which is required to compensate an extra degree of freedom of the non-degenerate massive vector compared to the eq. (4.23). Since the third term in (4.24) does not depend on curvature, we will omit it in what follows.

As we have just mentioned, the efficient check of the calculations can be done by taking the massless limit and consequent comparison with the eq. (3.10). However, for the vector case the massless limit can not be simple.[3] In fact, we have to link the massless limit of the expression (4.24) with the similar, but different formula for the massless vector:

$$\mathrm{Tr}\ln \hat{H}_v(m_v \equiv 0) = \mathrm{Tr}\ln \left(\delta^\mu_\nu \Box - R^\mu_\nu\right) - 2\,\mathrm{Tr}\ln \Box \,. \tag{4.25}$$

In the massless case, the original theory is gauge invariant and the formula (4.25) results from the DeWitt-Faddeev-Popov procedure. The second term in the *r.h.s.* of the last equation represent the contribution of the Faddeev-Popov gauge ghosts. Of course, there are two scalar (ghost) degrees of greedom in the gauge field case,[4] in contrast to the single compensating scalar in the massive vector theory. Hence, the receipt of how to check the expression for the effective gravitational action of a massive vector field is the following. First, one has to derive the contribution of both non-degenerate vector and minimal scalar fields and then use (4.23). But, in order to have a correspondence with the conformal anomaly in the massless limit, the scalar contribution must be multiplied by the factor of two.

Now we are in a position to perform the calculations. In the vector case

$$\hat{\mathcal{R}}_{\mu\nu} = \left[\hat{\mathcal{R}}_{\mu\nu}\right]^\alpha_\beta = -R^\alpha_{\beta\mu\nu} \,, \qquad \hat{P} = P^\nu_\mu = \frac{1}{6} R \delta^\nu_\mu - R^\nu_\mu \,. \tag{4.26}$$

[3]For instance, this can be seen from the third term in (4.24).

[4]As far as we are interested in the vacuum gravitational effect, there is no difference between abelian and non-abelian theories. For the sake of simplicity, we shall consider the simplest abelian version only.

After a simple algebra we obtain the following coefficients:

$$
\begin{aligned}
l_1 &= -\frac{1}{8}, \qquad l_2 = -\frac{1}{2}, \qquad l_3 = -\frac{1}{2}, \qquad l_4 = \frac{5}{12}, \qquad l_5 = \frac{1}{2}, \\
l_1^* &= \frac{1}{2}, \qquad l_2^* = 2, \qquad l_3^* = 4, \qquad l_4^* = -\frac{4}{3}, \qquad l_5^* = -4.
\end{aligned} \tag{4.27}
$$

Using (4.27), (4.27) and the eq. (4.4), we arrive at the effective action of a massive vector field

$$
\begin{aligned}
\bar{\Gamma}^{(1)}_{\text{vector}} &= \frac{1}{2(4\pi)^2}\int d^4x g^{1/2} \times \\
&\times \Big\{ \frac{3}{2} m_v^4 \cdot \left[\frac{1}{2-w} + \ln\left(\frac{4\pi\mu^2}{m_v^2}\right) + \frac{3}{2}\right] + \frac{1}{2} m_v^2 R \left[\frac{1}{2-w} + \ln\left(\frac{4\pi\mu^2}{m_v^2}\right) + 1\right] + \\
&+ \frac{1}{2} C_{\mu\nu\alpha\beta}\left[\frac{13}{60(2-w)} + \frac{13}{60}\ln\left(\frac{4\pi\mu^2}{m_v^2}\right) + k_W^v(a)\right] C^{\mu\nu\alpha\beta} + \\
&+ R\left[\frac{1}{72(2-w)} + \frac{1}{72}\ln\left(\frac{4\pi\mu^2}{m_v^2}\right) + k_R^v(a)\right] R \Big\},
\end{aligned} \tag{4.28}
$$

where

$$
\begin{aligned}
k_W^v(a) &= -\frac{91}{450} + \frac{2}{15a^2} - \frac{8A}{3a^2} + A + \frac{8A}{5a^4}, \\
k_R^v(a) &= -\frac{1}{2160} + \frac{A}{48} + \frac{A}{3a^4} + \frac{1}{36a^2} - \frac{A}{18a^2}.
\end{aligned} \tag{4.29}
$$

Here, a and A in eq. (4.11) depend on the vector mass m_v. The divergent part of (4.28) coincides with the expressions derived in [19]. It is easy to see that the massless limit of the formfactor $k_R^v(a)$ is singular, because the A-dependent terms do not cancel. This is exactly what we should expect, because the consistent massless limit requires subtracting one more scalar contribution. In case of the double compensating scalar, instead of (4.29), we meet the following formfactors in the higher derivative sector (here, index gv means "gauge-like vector")

$$
\begin{aligned}
k_W^{gv}(a) &= \frac{241}{3600} - \frac{5A}{16} - \frac{A}{5a^4} - \frac{1}{60a^2} + \frac{5A}{6a^2}, \\
k_R^{gv}(a) &= \frac{13}{1080} - \frac{A}{24} + \frac{1}{54a^2} + \frac{2A}{9a^4} + \frac{A}{9a^2}.
\end{aligned} \tag{4.30}
$$

Using these formfactors, in the massless limit $m_v \to 0$ we meet a non-singular result and obtain

$$
\bar{\Gamma}^{(1)}(m_v \to 0) = +\frac{1}{12\cdot 10(4\pi)^2}\int d^4x g^{1/2} R^2 + \cdots, \tag{4.31}
$$

in a perfect correspondence with (3.10).

It is clear that the last result can not be interpreted such that we achieved a non-singular massless limit of the Proca field contribution to the effective action. But, in this way we checked both non-degenerate vector and compensating scalar contributions and thus ensured the correctness of our result (4.28), (4.29).

JHEP06(2003)004

5. Renormalization group equations

The purpose of this section is to derive the renormalization group β-functions for the parameters a_1, a_3, and a_4 of the vacuum action (3.1) in the mass-dependent scheme. This calculation is relevant for the anomaly-induced inflation model of [4], because this model is based on the phenomena of the decoupling of the massive fields at low energies. Thus, we can not be completely satisfied by the standard mass-independent β-functions which arise in the Minimal Subtraction ($\overline{\mathrm{MS}}$) scheme. In fact, the $\beta^{(\overline{\mathrm{MS}})}$-functions describe the running only at high energies, when they correspond to the leading logarithms in the vacuum effective action. The corresponding expressions [28, 29] are the gravitational analogs of the generalized Euler-Heisenberg effective action in QED. An important advantage of the $\overline{\mathrm{MS}}$ scheme is that the renormalization group can be formulated in a completely consistent nonperturbative covariant way [30, 31, 32, 28, 8], while the renormalization of the vacuum action in the mass-dependent scheme is not so general and has to involve the covariant expansion in the curvature tensor or (as we did in the previous paper [1]) an expansion of the metric around the fixed (flat) background. However, in the high-energy UV limit the $\overline{\mathrm{MS}}$ scheme and the mass-dependent scheme β-functions must coincide and this enables one to check the envolved calculations in the mass-dependent scheme.

In the $\overline{\mathrm{MS}}$ scheme the β-function of the effective charge C is defined as

$$\beta_C(\overline{\mathrm{MS}}) = \lim_{n\to 4} \mu \frac{dC}{d\mu} \,. \tag{5.1}$$

The derivation of the β-functions for the parameters a_1, a_3, and a_4 of the vacuum action (3.1), in the mass-dependent scheme, has been described in [1] (see, e.g. [12] for the general technical introduction to the effective approach in Quantum Field Theory). On flat background, one has to subtract the counterterm at the momentum $p^2 = M^2$, where M is the renormalization point and calculate the β-function using the formula

$$\beta_C = \lim_{n\to 4} M \frac{dC}{dM} \,. \tag{5.2}$$

Instead, one can simply take the derivative $-p d/dp$ of the formfactors in the polarization operator. In the covariant formalism, we identify p^2 and $-\Box$ and rewrite the definition above using the variable a from the eq. (4.11), therefore, the β-functions are the operators in the x-space. Now we can apply this procedure to the formfactors of the $C^2_{\mu\nu\alpha\beta}$ and R^2 terms in the scalar, fermion, and vector cases.

5.1 Massive scalar

In this case the β_1-function has the form [1]

$$\beta_1^{\mathrm{scalar}} = -\frac{1}{(4\pi)^2}\left(\frac{1}{18a^2} - \frac{1}{180} - \frac{a^2-4}{6a^4}A\right), \tag{5.3}$$

that is the general result for the one-loop β-function valid at any scale. As one should expect, the β-function for the Weyl term coefficient a_1 does not depend on the nonminimal parameter ξ.

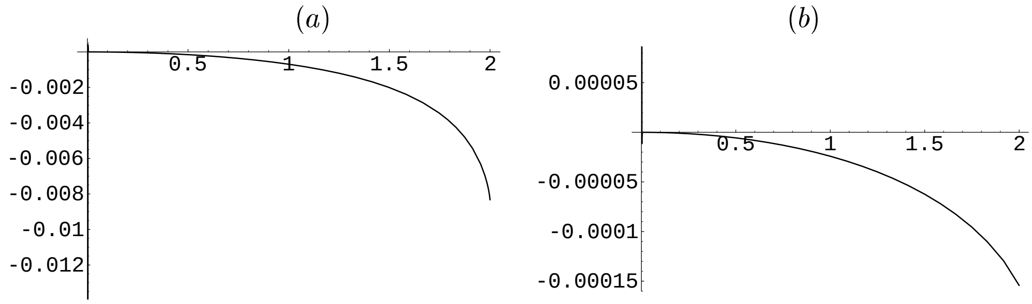

Figure 1: The plots of the β-functions (a) $\beta_1^{\rm scalar}$ and (b) $\beta_3^{\rm scalar}$ versus a. All the β-functions in this and consequent plots are presented in the units of $(4\pi)^2$.

As the special cases we meet the UV limit $p^2 \gg m^2$

$$\beta_1^{\rm scalar,UV} = -\frac{1}{(4\pi)^2}\frac{1}{120} + \mathcal{O}\left(\frac{m^2}{p^2}\right), \tag{5.4}$$

that agrees with the $\overline{\rm MS}$-scheme result (3.7). In the IR limit $p^2 \ll m^2$ we meet

$$\beta_1^{\rm scalar,IR} = -\frac{1}{1680(4\pi)^2}\cdot\frac{p^2}{m^2} + \mathcal{O}\left(\frac{p^4}{m^4}\right). \tag{5.5}$$

The last formula demonstrates the IR decoupling of the quantum effects of the massive scalar field. Moreover, the decoupling is a smooth monotone effect, that can be seen from the first plot (a) at the figure 1. In this plot as in all similar plots in other figures, we use the variable a defined in (4.11). The advantage of the use of a is that this variable changes from $a = 0$ in the IR to $a = 2$ in the UV, while p changes from $p = 0$ to $p = \infty$. It is important that the dependence $a^2 = 4p^2/(p^2 + 4m^2)$ on p^2 is also monotonic.

Let us consider the remaining β-functions β_3 and β_4. According to the definition given in section 3, β_4 is defined through the same procedure as β_1, if we separate the $(\xi - 1/6)$-proportional terms in the formfactor $k_R(a)$ in eq. (4.14). Direct calculations give the following expression (see also [1])

$$\beta_4^{\rm scalar} = -\frac{1}{(4\pi)^2}\left(\xi - \frac{1}{6}\right)\left\{\frac{4A - a^2A + a^2}{8}\cdot\left(\xi - \frac{1}{6}\right) + \frac{a^2-4}{48}\left(\frac{a^2-12}{a^2}A - 1\right)\right\}. \tag{5.6}$$

In the UV limit $p^2 \gg m^2$ the β-function is (in agreement with the standard $\overline{MS}$ result [31])

$$\beta_4^{{\rm scalar},UV} = -\frac{1}{2(4\pi)^2}\left(\xi - \frac{1}{6}\right)^2 + \mathcal{O}\left(\frac{m^2}{p^2}\right), \tag{5.7}$$

while in the IR limit $p^2 \ll m^2$ we obtain

$$\beta_4^{{\rm scalar},IR} = -\frac{1}{12(4\pi)^2}\left[\left(\xi - \frac{1}{6}\right)^2 - \frac{1}{15}\left(\xi - \frac{1}{6}\right)\right]\cdot\frac{p^2}{m^2} + \mathcal{O}\left(\frac{p^4}{m^4}\right). \tag{5.8}$$

In order to define the β_3-function, according to the definition given in the section 3, one do not need to take derivative Md/dM. Instead, we can use the definition

$$-\frac{2}{\sqrt{-g}}g_{\mu\nu}\frac{\delta\Gamma}{\delta g_{\mu\nu}} = \langle T^\mu_\mu\rangle\,,$$

and the relation between the quantum-corrected trace and the β-functions, which we generalize from the massless case.[5] Thus, the required β_3-function can be defined directly from the $(\xi - 1/6)$-independent part of the formfactor $k_R(a)$ in eq. (4.14):

$$\beta_3^{\text{scalar}} = \frac{1}{(4\pi)^2}\left[\frac{20-7a^2}{360a^2} + \frac{(a^2-4)^2 A}{24a^4}\right]. \tag{5.9}$$

The UV limit of this renormalization group function perfectly corresponds to the standard $\overline{MS}$ result (3.7)

$$\beta_3^{\text{scalar,UV}} = -\frac{1}{180(4\pi)^2} + \mathcal{O}\left(\frac{m^2}{p^2}\right), \tag{5.10}$$

while the IR limit demonstrates the decoupling of the scalar field

$$\beta_3^{\text{scalar,IR}} = -\frac{1}{1260(4\pi)^2}\frac{p^2}{m^2} + \mathcal{O}\left(\frac{p^4}{m^4}\right). \tag{5.11}$$

Finally (this is very important for the inflationary model of [3]), the decoupling goes smoothly and is monotonic, as one can see from the second plot at the figure 1.

5.2 Massive fermion

The β-function of the a_1 coefficient has the form

$$\beta_1^{\text{fermion}} = \frac{1}{(4\pi)^2}\left[\frac{2}{9a^2} - \frac{19}{180} + \left(\frac{8}{3a^4} - \frac{5}{3a^2} + \frac{1}{4}\right) A\right]. \tag{5.12}$$

The UV limit $p^2 \gg m^2$ gives

$$\beta_1^{\text{fermion,UV}} = -\frac{1}{20(4\pi)^2} + \mathcal{O}\left(\frac{m^2}{p^2}\right) \tag{5.13}$$

in agreement with the $\overline{\text{MS}}$-scheme result and also with the expression for divergences in the effective action (4.20). The IR limit $p^2 \ll m^2$ is qualitatively similar to the scalar case in the sense that it shows the decoupling

$$\beta_1^{\text{fermion,IR}} = -\frac{1}{168(4\pi)^2}\cdot\frac{p^2}{m^2} + \mathcal{O}\left(\frac{p^4}{m^4}\right). \tag{5.14}$$

The plot demonstrating the smooth and monotone form of the decoupling for the β_1 is the first one (a) at the figure 2.

According to our definition in section 3, the β_4-function is identically zero for the fermion field. The derivation of the β_3-function performs similarly to the scalar case, but for the fermions we have to use the formfactor $k_R^f(a)$ from (4.21). The result is

$$\beta_3^{\text{fermion}} = \frac{1}{(4\pi)^2}\left[\frac{a^2-10}{45a^2} + \frac{2(a^2-4)A}{3a^4}\right]. \tag{5.15}$$

[5]It is, of course, simpler to *define* the β_3-function as a $(\xi - 1/6)$-independent part of the R^2-sector of the effective action and do not mention the trace anomaly which plays only an illustrative role here. Our way of presentation is motivated by the will to maintain the link to the anomaly-induced effective action.

JHEP06(2003)004

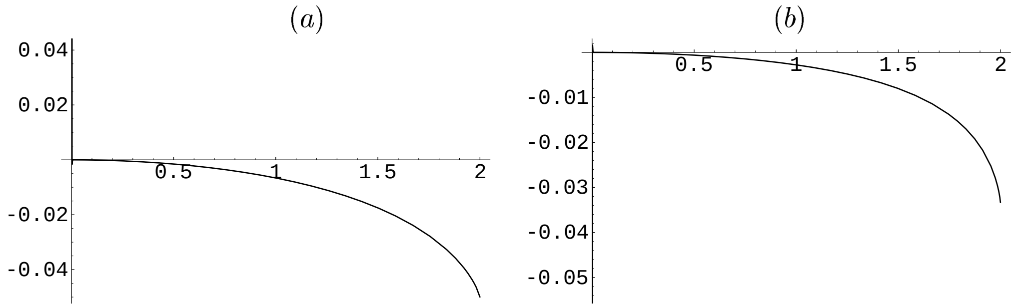

Figure 2: The plots of the β-functions (a) β_1^{fermion} and (b) β_3^{fermion} versus a.

The UV limit of this renormalization group function corresponds to the expected standard $\overline{MS}$ result (3.7)

$$\beta_3^{\text{fermion},UV} = -\frac{1}{30(4\pi)^2} + \mathcal{O}\left(\frac{m^2}{p^2}\right), \tag{5.16}$$

while the IR limit demonstrates the decoupling of the spinor field

$$\beta_3^{\text{fermion},IR} = -\frac{1}{420(4\pi)^2} \cdot \frac{p^2}{m^2} + \mathcal{O}\left(\frac{p^4}{m^4}\right). \tag{5.17}$$

The second plot at the figure 2 shows that the decoupling goes smoothly and is monotonic.

5.3 Massive vector

Despite the massless limit for the quantum vacuum corrections of the massive vector is singular (due to the gauge invariance and consequently different number of physical degrees of freedom), we can derive the general expressions for the β-functions using the usual receipt. Then, the β-function for the a_1 coefficient is

$$\beta_1^{\text{vector}} = \frac{1}{(4\pi)^2}\left[\frac{11}{60} - \frac{1}{6a^2} - \frac{a^2}{16} + \frac{(a^2-4)(a^4-8a^2+8)}{16a^4} \cdot A\right]. \tag{5.18}$$

The UV limit $p^2 \gg m^2$ demonstrates perfect correspondence with the divergent coefficient of the Weyl term in the effective action (4.28)

$$\beta_1^{\text{vector},UV} = -\frac{1}{(4\pi)^2}\frac{13}{120} + \mathcal{O}\left(\frac{m^2}{p^2}\right). \tag{5.19}$$

Of course, this corresponds, also, to the standard $\overline{\text{MS}}$-scheme result [19]. Compared to the UV limit, exactly as in the scalar and fermion cases, the IR regime $p^2 \ll m^2$ demonstrates the decoupling of the loop contribution

$$\beta_1^{\text{vector,IR}} = -\frac{3}{112(4\pi)^2} \cdot \frac{p^2}{m^2} + \mathcal{O}\left(\frac{p^4}{m^4}\right). \tag{5.20}$$

The decoupling occurs in a smooth manner, according to the first plot at the figure 3.

In the R^2- sector we meet a usual problem with the separation of the β_3 and β_4 functions. The problem is that, we can not use the same definition as for the scalar case, because

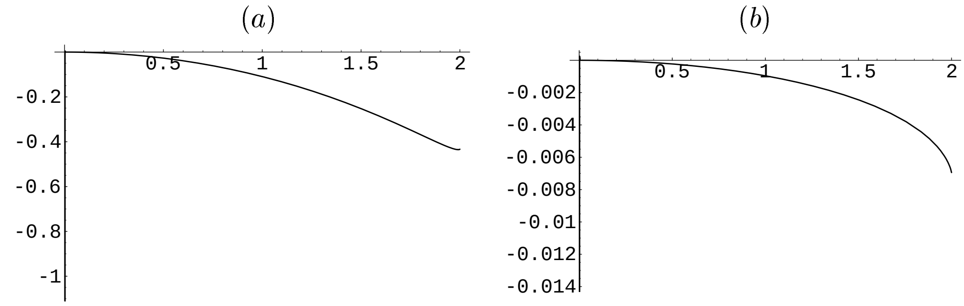

Figure 3: The plots of the β-functions (a) $\beta_1(a)$ and (b) $\beta_4(a)$ for the case of the massive vector field.

in the vector case there is no parameter similar to ξ. At the same time, the presence of the $\int \sqrt{-g}R^2$ divergence and the consequent UV singularity in the finite R^2- term (of course, this only means that the non-local renormalization-group related term (4.16) emerges) does not permit us to attribute all the R^2- term in the effective action to β_3. Hence, the explicit criterion for the separation is really absent. But, this does not indicate an inconsistency of the theory. Let us remind that the object of principal physical importance is the effective action (4.28), which has no ambiguity beyond the usual renormalization point dependence. The separation of the two β-functions means that we separate, in a certain manner, the finite and infinite R^2-terms. Then, fixing the ambiguity in the β-functions we define the certain framework for the well defined object such as effective action.

After we apply the standard procedure, the expression for the β_4-function has the form

$$\beta_4^{\text{vector}} = \frac{1}{768a^2(4\pi)^2}\left[\frac{(a^2-4)(80-8a^2+a^4)}{a^2}\cdot A - \frac{80-16a^2+3a^4}{3}\right]. \tag{5.21}$$

In the UV limit we meet

$$\beta_4^{\text{vector},UV} = -\frac{1}{144(4\pi)^2}\,, \tag{5.22}$$

in accordance with the corresponding divergence of the effective action (4.28), hence we agree with the $\overline{MS}$-scheme in this limit. In the IR limit the result is

$$\beta_4^{\text{vector,IR}} = -\frac{1}{1120(4\pi)^2}\cdot\frac{p^2}{m^2} + \mathcal{O}\left(\frac{p^4}{m^4}\right), \tag{5.23}$$

that demonstrates the usual decoupling. The plot of the β_3-function is presented at figure 3.

6. Decoupling in the case of supersymmetry

According to the analysis of the previous section, the β-functions show the standard IR behavior, that is the smooth and monotonic decoupling. It is interesting to see whether one can meet a similar feature of the β-functions in case of decoupling of the supersymmetric partners of the observable sector of the supersymmetric Standard Model (MSSM) or other supersymmetric gauge theory such as GUT. In the one-loop approximation, the

β-functions gain independent additive contributions from all the fields. We know that, for the energy scale below the mass of the particles, these β-functions start to decrease and for $p^2 \ll m^2$ the β-functions are quadratically suppressed. Therefore, in order to investigate the details of the superpartners decoupling, we need to know the masses of all these superpartners. Unfortunately, this information is not available, and hence there is no chance to perform the detailed quantitative analysis of the decoupling. At the same time, it is possible to construct a very simple model which can illustrate the principal characteristics of supersymmetry decoupling.

Let us consider, as an example, the minimal supersymmetric extension (MSSM) of the Standard Model (SM). The particle content of the SM includes 12 vectors (gluons, photon, W and Z vector bosons), quarks and leptons, which can be counted, in terms of the Dirac spinors, as $N_{quark} = 18$ and $N_{lepton} = 6$ (we suppose that the neutrino are Dirac massive particles, in case they are Majorana particles this number gets changed $N_{lepton} = 4.5$, but all the conclusions remain the same), a complex Higgs doublet, which is equivalent to four real scalars. In total, using the notations of (3.7), we have

$$N_0^{SM} = 4\,, \qquad N_{1/2}^{SM} = 24\,, \qquad N_1^{SM} = 12\,. \tag{6.1}$$

With this particle content we meet a positive overall sign of the induced $\int \sqrt{-g}R^2$-term in (3.10). The same sign takes place in the present-day Universe, where only photon is active. And this sign has a very strong physical meaning. If we do not introduce large negative a_4 coefficient in the classical action (3.1), the positive sign in (3.10) means non-stability of the anomaly-induced inflation [6]. On the contrary, if this sign is negative, the inflation is stable. Indeed, this happens in the MSSM, where

$$N_0^{MSSM} = 104\,, \qquad N_{1/2}^{MSSM} = 32\,, \qquad N_1^{MSSM} = 12\,. \tag{6.2}$$

As it was suggested in the first reference of [4], the decoupling of the sparticles may be responsible for the change of sign in (3.10), and this provides an appealing scheme of the modified Starobinsky model of inflation which starts in the stable regime due to supersymmetry and has a natural graceful exit to the FRW evolution after supersymmetry breaks down.

Our purpose is to obtain the qualitative description of the decoupling, without involving the details of the supersymmetric spectrum. Therefore, let us assume the simplest possible input. Suppose that, for some reason, *all* the sparticles have masses much larger than the masses of the observed particles.[6] For the sake of simplicity, we suppose that all the constituents of the SM as massless. Moreover, we shall simplify things further and suppose that all the sparticles have exactly the same mass, which we denote M^*. Hence, we arrive at the "super-simplified" model with the number of massless fields given by (6.1) and with the number of fields with an equal mass M^* given by the difference with (6.2). An additional advantage of our simplifications is that one does not have massive vectors in this model and hence we are free from the corresponding problems with their massless limit which have been discussed in sections 4 and 5.

[6]In some sense, this is a natural hypothesis. If there is no unknown general law of nature of this kind, it is very difficult to explain why the observed particles can not be superpartners of each other. Also, it is worth mentioning that this spectrum of supersymmetry leads to a natural inflation!

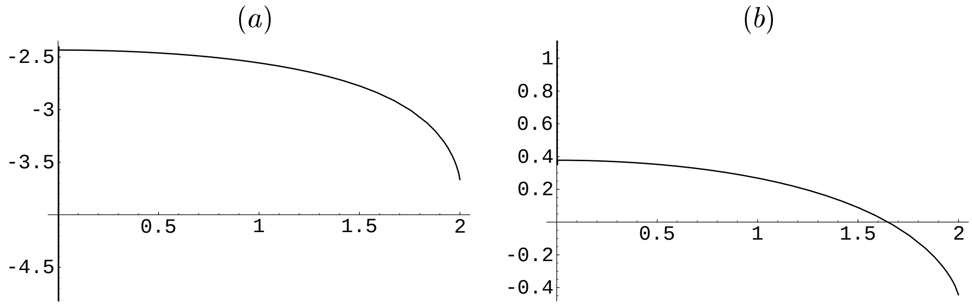

Figure 4: The plots of the β-functions (a) $\beta_1(a)$ and (b) $\beta_3(a)$ for the case of the "super-simplified" supersymmetry breaking model.

At energies higher than the EW scale, the massless SM fields contribute as

$$\beta_1^{SM} = N_0^{SM} \cdot \beta_1^{\text{scalar},\overline{\text{MS}}} + N_{1/2}^{SM} \cdot \beta_1^{\text{fermion},\overline{\text{MS}}} + N_1^{SM} \cdot \beta_1^{\text{vector},\overline{\text{MS}}}$$

and their massive superpartners as

$$\begin{aligned} \beta_1^{\text{SUSY}} &= (N_0^{MSSM} - N_0^{SM}) \cdot \beta_1^{\text{scalar}}(a) + (N_{1/2}^{MSSM} - N_{1/2}^{SM}) \cdot \beta_1^{\text{fermion}}(a) + \\ &\quad + (N_1^{MSSM} - N_1^{SM}) \cdot \beta_1^{\text{vector}}(a)\,, \end{aligned}$$

where the parameter a depends on the mass M^*. The overall β-function for the parameter a_1 is given by the sum

$$\beta_1^t = \beta_1^{SM} + \beta_1^{\text{SUSY}} = \frac{1}{(4\pi)^2}\left[\frac{2A(a^2-4)(3a^2+17)}{3a^4} - \frac{49a^2+68}{18a^2}\right]. \tag{6.3}$$

The UV and IR limits of this expression are given by

$$\begin{aligned} \beta_1^{tUV} &= -\frac{1}{(4\pi)^2}\cdot\frac{11}{3} + \mathcal{O}\left(\frac{m^2}{p^2}\right), \\ \beta_1^{tIR} &= -\frac{1}{(4\pi)^2}\cdot\left(\frac{73}{30} + \frac{3}{28}\frac{p^2}{m^2}\right) + \mathcal{O}\left(\frac{p^4}{m^4}\right) \end{aligned} \tag{6.4}$$

respectively, and the plot of the β-function is presented at the figure 4. As usual, the decoupling is smooth and monotone.

The β_3-function for the parameter a_3 can be derived in the very same way as it was explained in section 5. The final expression has the following form:

$$\beta_3^t = \frac{1}{(4\pi)^2}\frac{(25a^2-68)}{18a^4}\cdot\left[3A(a^2-4) - a^2\right], \tag{6.5}$$

The UV and IR limits of this expression are given by

$$\begin{aligned} \beta_3^{tUV} &= -\frac{1}{(4\pi)^2}\cdot\frac{4}{9} + \mathcal{O}\left(\frac{m^2}{p^2}\right), \\ \beta_3^{tIR} &= \frac{1}{(4\pi)^2}\cdot\left(\frac{17}{45} - \frac{31a^2}{315}\right) + \mathcal{O}\left(\frac{m^4}{p^4}\right). \end{aligned} \tag{6.6}$$

It is easy to see that the sign of the β_3-function changes from negative in the UV to positive in the IR, as we should (of course) expect. The β_3-function dependence on the momenta is smooth and monotonic, as can be observed at the second plot at the figure 4. We can see that the decoupling in the high derivative sectors goes in such a way that the transition between stable and unstable inflation [4] performs in a smooth way. One can expect a qualitatively similar behavior of the β-functions in realistic models of supersymmetry. In particular, this concerns all non-minimal extensions of the MSSM. Let us comment on this point. Since the supersymmetry is not observed at low energies, one can suppose that all sparticles have large masses. Moreover, the extra "nonminimal" sparticles are usually fermions and scalars, that corresponds to increasing the number of chiral superfields. Of course, one can increase the number of vector superfields also, but this would signify the introduction of a new fundamental force and hence the new model can not be treated as a "non-minimal extension" of the MSSM. Then, in the UV we meet $\beta_3 < 0$ and in the IR $\beta_3 > 0$, just as in the MSSM case. The smoothness of the β_3-function is guaranteed by the monotone behavior of the β_3-functions for scalars and fermions.

7. Conclusions

We have performed the calculations of the effective action of vacuum for the massive scalar, fermion, and vector fields to the second order in curvature, using a mass-dependent renormalization scheme. As a result, we have found the explicit form of the decoupling of the massive fields in the higher derivative sector of the vacuum effective action. In the high-energy limit there is a perfect correspondence between the β-functions derived in a mass-dependent scheme and the standard ones derived within the $\overline{MS}$-scheme. Also, in the same limit we have established the correspondence with the anomaly-induced effective action derived for the massless conformal fields.

In the low-energy limit the β-functions for the massive fields tend to zero in a smooth monotone way. For the supersymmetric model, the form of decoupling in the higher derivative sector indicates the possibility of the soft transition between the stable and unstable regimes in the anomaly-induced inflation [4]. In particular, we can observe explicitly, using the "super-simplified" (but reliable) supersymmetry breaking model, how the decoupling of sparticles occurs. The detailed investigation of the cosmological applications will be reported separately.

Acknowledgments

The authors are grateful to N. Berkovits for stimulating conversation. E.G. acknowledges warm hospitality of the Physics Department at the Federal University of Juiz de Fora. The work of the authors has been supported by the research grant from FAPEMIG and (I.Sh.) by the fellowship from CNPq.

References

[1] E.V. Gorbar and I.L. Shapiro, *Renormalization group and decoupling in curved space*, *J. High Energy Phys.* **02** (2003) 021 [hep-ph/0210388].

[2] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111].

[3] J.C. Fabris, A.M. Pelinson and I.L. Shapiro, *On the gravitational waves on the background of anomaly induced inflation Nucl. Phys.* **B 597** (2001) 539.

[4] I.L. Shapiro, *The graceful exit from the anomaly-induced inflation: supersymmetry as a key*, *Int. J. Mod. Phys.* **D11** (2002) 1159–1170 [hep-ph/0103128];
I.L. Shapiro and J. Sola, *Massive fields temper anomaly-induced inflation*, *Phys. Lett.* **B 530** (2002) 10 [hep-ph/0104182];
A.M. Pelinson, I.L. Shapiro and F.I. Takakura, *On the stability of the anomaly induced inflation*, *Nucl. Phys.* **B 648** (2003) 417.

[5] M.V. Fischetti, J.B. Hartle and B.L. Hu, *Quantum effects in the early universe, I. Influence of trace anomalies on homogeneous, isotropic, classical geometries*, *Phys. Rev.* **D 20** (1979) 1757.

[6] A.A. Starobinsky, *A new type of isotropic cosmological models without singularity*, *Phys. Lett.* **B 91** (1980) 99;
A. Vilenkin, *Classical and quantum cosmology of the starobinsky inflationary model*, *Phys. Rev.* **D 32** (1985) 2511;
P. Anderson, *Effects of quantum fields on singularities and particle horizons in the early universe*, *Phys. Rev.* **D 28** (1983) 271; *Effects of quantum fields on singularities and particle horizons in the early universe, II*, *Phys. Rev.* **D 29** (1984) 615; *Effects of quantum fields on singularities and particle horizons in the early universe, IV. Initially empty universes*, *Phys. Rev.* **D 29** (1986) 1567.

[7] N.D. Birell and P.C.W. Davies, *Quantum fields in curved space*, Cambridge University Press, Cambridge 1982.

[8] I.L. Buchbinder, S.D. Odintsov and I.L. Shapiro, *Effective action in quantum gravity*, IOP Publishing, Bristol 1992.

[9] T. Appelquist and J. Carazzone, *Infrared singularities and massive fields*, *Phys. Rev.* **D 11** (1975) 2856.

[10] I.L. Shapiro and J. Sola, *On the scaling behavior of the cosmological constant and the possible existence of new forces and new light degrees of freedom*, *Phys. Lett.* **B 475** (2000) 236 [hep-ph/9910462];
I.L. Shapiro and J. Sola, *Scaling behavior of the cosmological constant: interface between quantum field theory and cosmology*, *J. High Energy Phys.* **02** (2002) 006 [hep-th/0012227].

[11] J.C. Collins, *Renormalization*, Cambridge Univiversity Press, Cambridge, 1984.

[12] A.V. Manohar, *Effective field theories*, hep-ph/9606222.

[13] K. Hagiwara et al. (Particle Data Group), *Review of particle physics. Particle data group Phys. Rev.* **D 66** (2002) 010001;
http://pdg.lbl.gov.

[14] J.J. Sakurai, *Currents and Methods*, University of Chicago, Chicago, 1969.

[15] G. Ecker, *Chiral symmetry*, hep-ph/9805500.

[16] S. Weinberg, *Phenomenological lagrangians*, *Physica* **96A** (1979) 327.

[17] J. Gasser and H. Leutwyler, *Chiral perturbation theory to one loop*, *Ann. Phys. (NY)* **158** (1984) 142; *Chiral perturbation theory: expansions in the mass of the strange quark*, *Nucl. Phys.* **B 250** (1985) 465.

[18] D.G. Boulware, *Renormalizability of massive non-abelian gauge fields: a functional integral approach*, *Ann. Phys. (NY)* **56** (1970) 140.

[19] A.O. Barvinsky and G.A. Vilkovisky, *The generalized Schwinger-DeWitt technique in gauge theories and Quantum Gravity*, *Phys. Rept.* **119** (1985) 1.

[20] A.O. Barvinsky and G.A. Vilkovisky, *Beyond the Schwinger-Dewitt technique: converting loops into trees and in-in currents Nucl. Phys.* **B 282** (1987) 163.

[21] R.J. Riegert, *A nonlocal action for the trace anomaly*, *Phys. Lett.* **B 134** (1984) 56.

[22] E.S. Fradkin and A.A. Tseytlin, *Conformal anomaly in weyl theory and anomaly free superconformal theories*, *Phys. Lett.* **B 134** (1984) 187.

[23] I.G. Avramidi, *Covariant studies of nonlocal structure of effective action*, *Yad. Fiz.* **49** (1989) 1185.

[24] F. Bastianelli and A. Zirotti, *Worldline formalism in a gravitational background*, *Nucl. Phys.* **B 642** (2002) 372;
F. Bastianelli, O. Corradini and A. Zirotti, *Dimensional regularization for SUSY sigma models and the worldline formalism*, *Phys. Rev.* **D 67** (2003) 104009 [hep-th/0211134].

[25] L.S. Brown and J.P. Cassidy, *Stress tensor trace anomaly in a gravitational metric: scalar fields*, *Phys. Rev.* **D 15** (1977) 1469; *Stress tensor trace anomaly in a gravitational metric: general theory, maxwell field*, *Phys. Rev.* **D 15** (1977) 2810.

[26] S.J. Hathrell, *Trace anomalies and lambda phi**4 theory in curved space*, *Ann. Phys. (NY)* **139** (1982) 136;
Trace anomalies and QED in curved space, *Ann. Phys. (NY)* **142** (1982) 34.

[27] G. De Berredo-Peixoto, *A note on the heat kernel method applied to fermions*, *Mod. Phys. Lett.* **A 16** (2001) 2463 [hep-th/0108223].

[28] I.L. Buchbinder and J.J. Wolfengaut, *Renormalization group equations and effective action in curved space-time*, *Class. and Quant. Grav.* **5** (1988) 1127.

[29] A. Dobado and A.L. Maroto, *Particle production from nonlocal gravitational effective action*, *Phys. Rev.* **D 60** (1999) 104045.

[30] I.L. Buchbinder, *Renormalization group equations in curved space-time*, *Theor. Math. Phys.* **61** (1984) 393.

[31] B.L. Nelson and P. Panangaden, *Scaling behavior of interacting quantum fields in curved space-time*, *Phys. Rev.* **D 25** (1982) 1019; *Universality and quantum gravity*, *Phys. Rev.* **D 29** (1984) 2759 see also [32] for the case of vector fields and [30] and [33] for the general fomulation of the renormalization group in curved space-time.

[32] D.J. Toms, *The background - field method and the renormalization of nonabelian gauge theories in curved space-time*, *Phys. Rev.* **D 27** (1983) 1803.

[33] D.J. Toms, *The effective action and the renormalization group equation in curved space-time*, *Phys. Lett.* **B 126** (1983) 37;
L. Parker and D.J. Toms, *Renormalization group analysis of grand unified theories in curved space-time*,*Phys. Rev.* **D 29** (1984) 1584.

JHEP06(2003)004